Session 1: A demand system for the cross-section of stocks

Introduction

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Structure of the course

- Lectures take place on May 6, May 13, May 20, and May 28.
- There are three problem sets to familiarize you with the data, model estimation, and counterfactuals.
- You can post questions in the chat, which will be monitored by one of us.
- Feel free to follow up by email if you have questions: `myogo@princeton.edu` / ralph.koijen@chicagobooth.edu / `xgabaix@fas.harvard.edu`.
- We will not record the lectures to have an open discussion about the topics discussed in this course.

Agenda

1. Session 1: A demand system for the cross-section of stocks.
   - Topics: Micro foundations, data construction, identification, estimation, and applications.
2. Session 2: A demand system for the cross-section of global financial markets.
   - Topics: Micro foundations, data construction, identification, estimation, and applications.
3. Session 3: A dynamic asset demand system for the aggregate market and the cross-section of stocks.
   - Topics: A dynamic asset demand system, identification, and estimation.
4. Session 4: Asset embeddings and open questions.
   - Topics: Using AI methods to extract asset embeddings from the asset demand system and open research questions.

Modern approaches to asset pricing

- Much of asset pricing evolves around models of the stochastic discount factor (SDF, "M").
- Broadly speaking, there are four classes of models:
  1. Empirical models with traded factors.
     E.g., Fama and French, Hou, Xue, and Zhang, Asness, Moskowitz, and Pedersen, as well as much of the recent machine-learning literature.

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- Econometric tests connect asset prices to the model's state variables or their innovations (e.g., Euler equation tests).

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     - Solution: Factor models and characteristics-based demand.
  3. Limited econometric tools to identify demand elasticities.
     - Unstable/unidentified estimates or impose mean-variance preferences to capture substitution patterns (Frankel, 1985).
     - Solution: Creative new instruments have been proposed in recent years.

Connecting the SDF and demand system approaches

- Any asset pricing model that starts from preferences, beliefs, . . . , implies
  1. An SDF that can be used to price assets using $\mathbb{E}[MR] = 1$.
  2. A demand system, $Q_i(P)$, that can be used to price assets by imposing market clearing, $\sum_i Q_i(P) = S$.

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- Throughout this course, we will highlight the implications of the same model for the asset demand system and the SDF, and thus how those are connected.

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- To provide credible quantitative answers to these questions, we need a well-specified asset demand system.
- See here for a detailed discussion.

Demand elasticities in standard asset pricing models

- We first compare our priors to asset pricing theory and then review the empirical evidence.
- Asset pricing theories generally imply downward-sloping demand.
  - Risk aversion, intertemporal hedging demand (Merton, 1973), price impact (Wilson, 1979 and Kyle, 1989).
- It is a quantitative question: What is the slope of the demand curve?
- Let us consider a standard CAPM calibration following Petajisto (2009) to fix ideas.

Demand elasticities in standard asset pricing models

CARA - normal model:

- N stocks with supply u_n each.
- Risk-free rate with infinitely-elastic supply, normalized to 0.
- Liquidating dividend for stock n

$$X_n = a_n + b_n F + e_n,$$

where F is the common factor and e_n the idiosyncratic risk.

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- There exists a continuum of investors that aggregate to a representative consumer with CARA preferences

$$\max_{\theta_i} E[-\exp(-\gamma W)], \qquad W = W_0 + \sum_{n=1}^{N} \theta_n (X_n - P_n).$$

Demand elasticities in standard asset pricing models

- Solving for equilibrium demand and set it equal to supply, u_n

$$P_n = a_n - \gamma \left[\sigma_m^2 \left(\sum_{m \neq n} u_m b_m \right) b_n + (\sigma_m^2 b_n^2 + \sigma_e^2) u_n \right].$$

The price discount will be dominated by the first term, not supply (the second term).

Demand elasticities in standard asset pricing models

- Calibration
  - $N = 1000$, $a_i = 105$, $b_i = 100$, $\sigma_e^2 = 900$, $\sigma_m^2 = 0.04$, $u_i = 1$, $\gamma = 1.25 \times 10^{-5}$.
    $\Rightarrow$ Market risk premium equals 5%, all stocks have a price of 100, a market beta of 1, and a standard deviation idiosyncratic risk of 30%.
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  - A supply shock of -10% to a stock: $u_n = 0.9$ for one stock.
  - The price of the stock increases by 0.16bp.
  - Part of this increase is due to the reduction in the aggregate market risk premium as there is less aggregate risk $\Rightarrow$ All stocks increase by 0.05bp.
  - Hence, the differential impact is only 0.11bp. This is what we mean with virtually flat demand curves.
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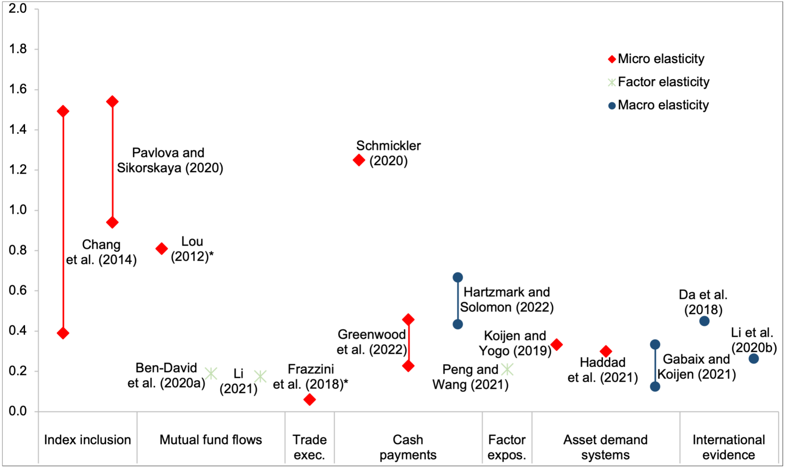
  - Intuitively, stocks are just very close substitutes. What matters most is a stock's beta and its contribution to aggregate risk.
  - Price elasticity of demand: $-\frac{\Delta Q/Q}{\Delta P/P} = \frac{0.10}{0.000016} \simeq 6,250$.
  - (Multiply by the dividend yield in dynamic models).

Micro versus macro elasticities

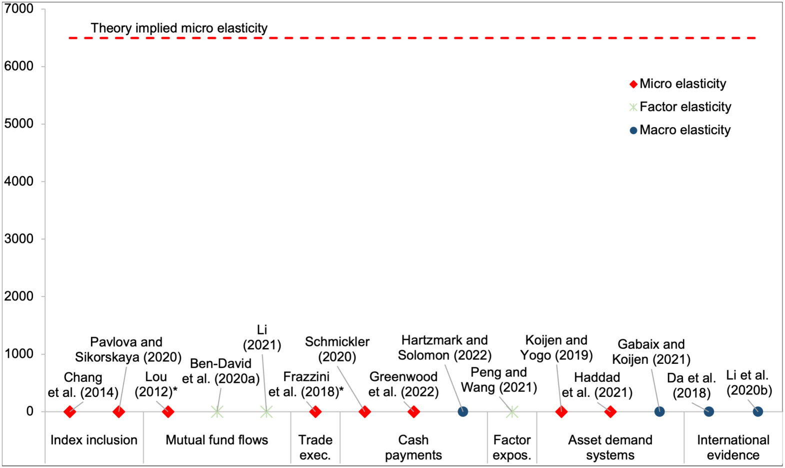
- Most of the literature focuses on individual securities (stocks, bonds, ...).
- This measures a micro elasticity.
- When aggregating to higher levels, such as factors (e.g., size and value) and the market, elasticities fall in standard models.
- Intuitively, two bio-tech firms are closer substitutes than stocks and bonds.
- See Gabaix and Koijen (2023) for an analysis of the macro elasticity.
  - In modern macro-finance models, the macro elasticity is around 20 $\Rightarrow$ More than 10 times larger compared to the empirical estimates for the micro elasticity.

Empirical evidence on demand elasticities



Source: Gabaix and Koijen (2023)

Empirical evidence on demand elasticities vs micro theory



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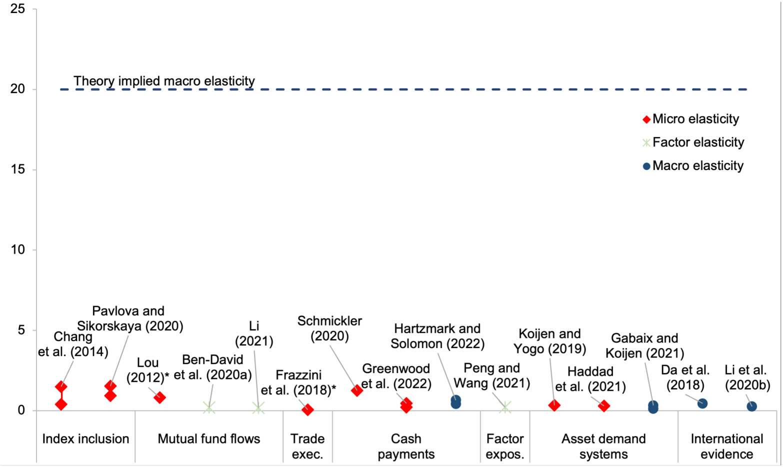

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Towards an empirically-tractable model of demand

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- Standard mean-variance portfolio choice implies

$$w = \frac{1}{\gamma}\Sigma^{-1}\mu.$$

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- Key insight: Solution simplifies under realistic assumptions to

$$w(n) = \frac{b'x(n)}{c},$$

where c encodes the information of all other stocks.

Various micro-foundations lead to a demand system

- Various micro-foundations.
  - Mean-variance portfolio choice (Markowitz 1952).
  - Portfolio choice with hedging demand (Merton 1973).
  - Private information and imperfect competition (Kyle 1989).
  - Heterogeneous beliefs.
  - Institutional asset pricing with constraints.
  - Direct preferences for characteristics such as ESG.
- Can be expressed as the same portfolio demand function (see KRY23).
- However, demand elasticities depend on structural parameters in different ways.

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- We have $i = 1, \ldots, I_x$, $x = Q, F$, investors of each type.
- Investors have CARA preferences

$$\max_{\mathbf{q}_i} \mathbb{E}\left[- \exp\left(-\gamma_i A_{1i}\right)\right],$$

with risk aversion coefficients $\gamma_i = \frac{1}{\tau_i A_{i0}}$ and initial assets A_{i0}.

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  with risk aversion coefficients $\gamma_i = \frac{1}{\tau_i A_{i0}}$ and initial assets A_{i0}.
- Investors allocate capital to $n = 1, \ldots, N$ assets.
- Intra-period budget constraint:

$$A_{0i} = \boldsymbol{q}_i' \boldsymbol{P}_0 + Q_i^0,$$

- Dividends are given by $\boldsymbol{D}_1$, which equal $\boldsymbol{P}_1$ in a static model.

Beliefs: Quant investors (KY19)

- Let $\boldsymbol{R}_1 = \boldsymbol{P}_1 - \boldsymbol{P}_0$ be the (dollar) return.
- Quants reason in terms of factor models and try to discover alpha as a function of asset characteristics

$$\begin{aligned} \boldsymbol{R}_1 &= \boldsymbol{a}_i + \boldsymbol{\beta}_i R_1^m + \boldsymbol{\eta}_1, \\ \boldsymbol{\mu}_i &= \boldsymbol{\alpha}_i + \boldsymbol{\beta}_i \Lambda, \end{aligned}$$

  where $\boldsymbol{\mu}_i = \mathbb{E}_i[\boldsymbol{R}_1]$ and $\mathrm{Var}(\boldsymbol{\eta}_1) = \sigma^2 \boldsymbol{I}$.
- Hence, the covariance matrix of returns is

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- Key: Alphas and betas are affine in characteristics,

$$\begin{aligned} \beta_i(n) &= \boldsymbol{\lambda}_i^{\beta\prime} \boldsymbol{x}(n) + \nu_i^{\beta}(n), \\ \alpha_i(n) &= \boldsymbol{\lambda}_i^{\alpha\prime} \boldsymbol{x}(n) + \nu_i^{\alpha}(n). \end{aligned}$$

Beliefs: Fundamental investors (KRY23)

- Let $\boldsymbol{R}_1^F = \boldsymbol{D}_1 - \boldsymbol{P}_0$ be the long-run fundamental return.
- Fundamental investors think about the long-run expected growth rate of fundamentals and their riskiness

$$\boldsymbol{D}_1 = \boldsymbol{g}_i + \boldsymbol{\rho}_i F_1 + \boldsymbol{\epsilon}_1,$$

where $\mathrm{Var}(\boldsymbol{\epsilon}_1) = \sigma^2 \boldsymbol{I}$.

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- Key: Factor loadings and expected growth are affine in characteristics,

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Demand curves

- The quant's optimal portfolio is

$$\boldsymbol{q}_i^Q = \frac{1}{\gamma_i} \boldsymbol{\Sigma}_i^{-1} \boldsymbol{\mu}_i.$$

- The optimal portfolio of the fundamental investor is

$$\boldsymbol{q}_i^F = \frac{1}{\gamma_i} \left(\boldsymbol{\Sigma}_i^F \right)^{-1} (\boldsymbol{g}_i - \boldsymbol{P}_0).$$

Key insight

- In both cases, the demand curve takes the form

$$\boldsymbol{q}_i = \frac{1}{\gamma}\left(\boldsymbol{v}_i\boldsymbol{v}_i' + \sigma^2\boldsymbol{I}\right)^{-1}\boldsymbol{m}_i.$$

Key insight

- In both cases, the demand curve takes the form

$$\boldsymbol{q}_i = \frac{1}{\gamma}\left(\boldsymbol{v}_i\boldsymbol{v}_i' + \sigma^2 \boldsymbol{I}\right)^{-1}\boldsymbol{m}_i.$$

- Using the Woodbury matrix identity, we have

$$\begin{aligned}\boldsymbol{q}_i &= \frac{1}{\gamma\sigma^2}\left(\boldsymbol{I} - \frac{\boldsymbol{v}_i\boldsymbol{v}_i'}{\boldsymbol{v}_i'\boldsymbol{v}_i + \sigma^2}\right)\boldsymbol{m}_i \\ &= \frac{1}{\gamma\sigma^2}\left(\boldsymbol{m}_i - c_i\boldsymbol{v}_i\right),\end{aligned}$$

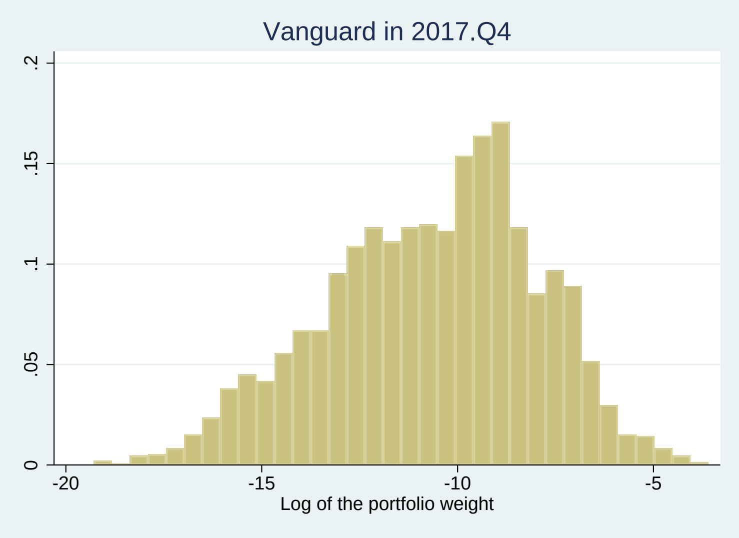
  where $c_i = \frac{\boldsymbol{v}_i'\boldsymbol{m}_i}{\boldsymbol{v}_i'\boldsymbol{v}_i + \sigma^2}$ is a scalar that encodes the information of all other stocks.
- The demand for stock n only depends on the characteristics of stock n and a common scalar, c_i.
- Intuition: The factor exposure and alpha are sufficient statistics for the attractiveness of stock n.

Three implementations of the mean-variance portfolio

- Estimate mean-variance portfolio among stocks in the S&P 500 index, subject to short-sale constraints.
  1. Benchmark: Unrestricted mean and covariance matrix.
  2. Factor structure: Impose FF 5-factor model on mean and covariance.
  3. Characteristics: Exponential-linear function of characteristics.

Statistic	Benchmark	Factor structure	Characteristics
Mean (%)	1.1	1.5	1.5
Standard deviation (%)	4.3	6.2	5.9
Certainty equivalent (%)	1.0	1.3	1.3
Correlation:			
Factor structure	0.54		
Characteristics	0.50	0.93	

Empirical regularity: Holdings are log-normally distributed



An empirically tractable asset demand system

- Investors select stocks in a choice set $\mathcal{N}_i \subset \{1, \ldots, N\}$.
- The portfolio weight on stock n is

$$w_i(n) = \frac{\delta_i(n)}{1 + \sum_{m \in \mathcal{N}_i} \delta_i(m)},$$

where

$$\delta_i(n) = \exp(b_{0,i} + \beta_{0,i} me(n) + \beta_{1,i}' x(n)) \epsilon_i(n).$$

and

- $b_{0,i}$: Controls the fraction invested in the outside asset.
- $\beta_{0,i} < 1$: Controls the price elasticity of demand.
- $me(n)$: Log market equity.
- $x(n)$: Stock characteristics (e.g., log book equity, profitability).
- $\beta_{1,i}$: Demand for characteristics.
- $\epsilon_i(n) \geq 0$: Latent demand.

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- A passive portfolio using market weights is replicated by
  - $\beta_{0,i} = 1$
  - $\beta_{1,i} = 0$
  - $\epsilon_i(n) = 1$.

Solve for asset prices by imposing market clearing

- Market clearing

$$ME(n) = \sum_{i=1}^{I} A_i w_i(n, \mathbf{me}, \mathbf{x}, \epsilon).$$

- KY19 show that a unique equilibrium exists if demand is downward sloping for all investors (i.e., $\beta_{0,i} < 1$).
- Despite this high-dimensional, nonlinear system in asset prices, we will discuss a simple algorithm to solve it quickly.

Lessons learned

- Assumptions commonly made in empirical asset pricing,
  1. Factor loadings depend on characteristics,
  2. Alphas depend on characteristics,

  have a convenient implication for optimal portfolios.
- Optimal demand for stock n only depends on that stock's characteristics and a scalar that encodes the information of all other stocks.
- We introduced an empirically-tractable model of the demand curve that adopts this structure and matches the lognormal property of portfolio weights.

Data sources US equities

- In KY19, we use the following data sources:
  - Prices and shares outstanding: CRSP.
  - Accounting data: Compustat.
  - Holdings data: 13-F filings accessed via Thomson-Reuters (S34).
- Alternative sources for 13-F filings:
  - Thomson Reuters Ownership.
  - FactSet Ownership (used in KRY23).

Data construction: Holdings data

- SEC Form 13F is the primary source: Quarterly stock holdings of institutions managing over $100m.
- Several notes:
  - 13F data are at the level of the institution (e.g., Vanguard instead of the Vanguard Small Cap Value Index Fund).
  - The filings are due 45 days after the end of the quarter.
  - Those filings can be restated later in case the earlier filings contained mistakes or some holdings were marked as confidential.
  - Form 13F reports only long positions and not short positions.
  - Cash and bond positions are not reported.
- The data are merged on CUSIP with the CRSP-Compustat data.

Investor types

- Thomson-Reuters provides type codes.
- Unfortunately, those contain mistakes in S34 since the late nineties.
- We fix those in KY19 and assign institutions to:
  - Banks.
  - Insurance companies
  - Investment advisors.
  - Mutual funds.
  - Pension funds.
  - Other 13F institutions (e.g., endowments, foundations, and nonfinancial corporations).
- FactSet also provides consistent type codes, also identifying hedge funds.

Investment universe

- Empirically, we find that investors hold few stocks and that this set is fairly stable over time.
- We construct the "investment universe," $\mathcal{N}_{it}$, which are investor-level sets of stocks that the investor can hold, even though the actual weight may be zero in a given quarter.
- Stocks outside the investment universe, $n \notin \mathcal{N}_{it}$, always receive a weight of zero.
- To construct the investment universe, we include all stocks held in the current quarter and the previous k quarters.
  - KRY23 show robustness when choosing the window, either further back or also forward.

Facts about holdings: Persistence of holdings

AUM	Previous quarters										
percentile	1	2	3	4	5	6	7	8	9	10	11
1	82	85	86	88	89	90	91	92	93	93	94
2	85	87	89	91	92	92	93	94	94	95	95
3	85	88	89	90	91	92	93	93	94	94	95
4	85	87	89	90	91	92	92	93	93	94	94
5	85	87	89	90	90	91	92	92	93	93	94
6	85	87	88	89	90	91	92	92	93	93	94
7	84	86	88	89	90	91	91	92	92	93	93
8	84	87	88	90	90	91	92	92	93	93	94
9	87	89	90	91	92	93	93	94	94	94	95
10	92	93	94	95	95	96	96	96	97	97	97

Facts about holdings

			Assets under management ($ million)		Number of stocks held		Number of stocks in investment universe	
Period	Number of institutions	% of market held	Median	90th prctile	Median	90th prctile	Median	90th prctile
1980–'84	544	35	337	2,666	118	386	183	523
1985–'89	780	41	400	3,604	116	451	208	691
1990–'94	979	46	404	4,563	106	511	192	810
1995–'99	1,319	51	465	6,579	102	555	176	942
2000–'04	1,801	57	371	6,095	88	520	165	982
2005–'09	2,443	65	333	5,424	73	460	145	922
2010–'14	2,883	65	315	5,432	67	445	122	798
2015–'17	3,664	67	301	5,186	67	451	111	743

Institutional holdings data in the United States

- Equity:
  - Mutual funds and ETFs: Morningstar and FactSet.
- Fixed income:
  - Mutual funds and ETFs: Morningstar and FactSet.
  - Insurance companies: Schedule D (NAIC, SNL, AM Best).
  - Refinitiv's eMAXX combines various sources.

International holdings data

- Securities Holdings Statistics.
  - Compiled by the ECB based on custodial records.
  - Securities-level data by country and sector.
- IMF Coordinated Investment Portfolio Statistics (CPIS).
  - Country-level cross-country holdings of short-term bonds, long-term bonds, and equity.
- Treasury International Capital (TIC) System.
  - Domestic and foreign holdings of US assets.
  - US holdings of foreign assets.

Household-level data

- So far, households are constructed as the residual of institutional holdings.
- In various countries, direct data on holdings are available.
  - US brokerage data (Barber and Odean 2000).
  - Statistics Sweden (Calvet et al. 2007).
  - Norwegian Central Securities Depository (Betermier et al. 2022).
  - Also Brazil, China, and India.
- These data can be used to unbundle the household sector and explore the implications of aggregation.
- For U.S. data, Gabaix, Koijen, Mainardi, Oh, and Yogo (2023) use data from Addepar to analyze demand of high net worth households.

Summary

- In many markets, detailed data on holdings are available.
- Regulators or supervisors, typically have additional data that can potentially be accessed.
- Most of those markets have not yet been explored, which creates unique research opportunities.

Identification and estimation of asset demand systems

Two central issues in asset demand estimation:

1. Latent demand is jointly endogenous with asset prices.
   - This is true when some investors are large or when latent is correlated across investors.
   - We need an instrument to estimate the model.
2. Implementation choices.
   - Some investors hold concentrated portfolios.
   - How to handle zero holdings in investors' portfolios.

Empirical specification

- Our model for demand

$$w_i(n) = \frac{\delta_i(n)}{1 + \sum_{m \in \mathcal{N}_i} \delta_i(m)},$$

implies for the fraction invested in the outside asset

$$w_i(0) = 1 - \sum_{n \in \mathcal{N}_i} w_i(n) = \frac{1}{1 + \sum_{m \in \mathcal{N}_i} \delta_i(m)}.$$

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- Combining both equations implies

$$\frac{w_i(n)}{w_i(0)} = \delta_i(n) = \exp(b_{0,i} + \beta_{0,i} me(n) + \beta_{1,i}' \mathbf{x}(n)) \epsilon_i(n).$$

Empirical specification

- Given an instrument for market cap, $\widehat{me}_i(n)$, we can estimate the model in two ways:

  1. Nonlinear GMM (with zero weights).

$$\frac{w_i(n)}{w_i(0)} = \exp(b_{0,i} + \beta_{0,i} me(n) + \beta_{1,i}' \mathbf{x}(n)) \epsilon_i(n)$$

  - Moment condition: $\mathbb{E}[\epsilon_i(n) | \widehat{me}_i(n), \mathbf{x}(n)] = 1$.

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2. Linear IV (without zero weights).

$$\log\left(\frac{w_i(n)}{w_i(0)}\right) = b_{0,i} + \beta_{0,i} me(n) + \beta_{1,i}' \mathbf{x}(n) + \log(\epsilon_i(n))$$

   - Moment condition: $\mathbb{E}[\log(\epsilon_i(n)) | \widehat{me}_i(n), \mathbf{x}(n)] = 0.$

Empirical specification

- Characteristics.
  1. Log book equity.
  2. Profitability.
  3. Investment.
  4. Dividends to book equity.
  5. Market beta.
- For each 13F institution and the household sector, use the cross-section of holdings to estimate coefficients at each point in time.
- Traditional assumption in endowment economies:

$$\mathbb{E}[\epsilon_i(n) | me(n), \mathbf{x}(n)] = 1$$

The role of the outside asset in estimation

- When we include investor-quarter fixed effects in the specification, a_{it}, the choice of the outside asset does not matter for estimation:

$$\log\left(\frac{w_{it}(n)}{w_{it}(0)}\right) = a_{it} + \beta_{0,it} me_t(n) + \beta_{1,it}' \mathbf{x}_t(n) + \log(\epsilon_{it}(n)).$$

- Any choice of $w_{it}(0)$ will be absorbed in a_{it} and we can equivalently estimate:

$$\begin{aligned}\log(w_{it}(n)) =& (a_{it} + \log(w_{it}(0)) \\ & + \beta_{0,it} me_t(n) + \beta_{1,it}' \mathbf{x}_t(n) + \log(\epsilon_{it}(n)).\end{aligned}$$

- The choice of the outside asset will matter in counterfactuals.

Identification

- Latent demand is generally correlated with prices.
  - Mechanically true if an investor is large.
  - Even with a continuum of investors, if there are common components in latent demand (e.g., sentiment, news media, corporate events, ...), then latent demand and prices are correlated.
- We therefore need an instrument for market equity.
- Before discussing specific instruments, we develop some intuition for where to find candidate instruments.

Identification: Intuition

- Portfolio weight for a group of investors indexed by g (omitting constants)

$$w_g(n) = \beta_{0,g} ME(n) + \lambda_g \eta(n) + u_g(n),$$

where $u_g(n)$ is uncorrelated across groups of investors.

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- Market clearing implies $\sum_g A_g w_g(n) = ME(n)$ and thus

$$ME(n) = \frac{\lambda_S \eta(n) + u_S(n)}{1 - \beta_{0,S}},$$

where $x_S = \frac{\sum_g A_g x_g}{\sum_g A_g}$, the size-weighted average.

Identification: Intuition

- Substitute the market clearing price into the demand equation

$$w_g(n) = \left(\lambda_g + \frac{\beta_{0,g}}{1-\beta_{0,S}}\lambda_S\right)\eta(n) + \frac{\beta_{0,g}}{1-\beta_{0,S}}u_S(n) + u_g(n).$$

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- Key insights:
  1. Common demand shocks, $\eta(n)$, cannot be used to identify elasticities. We can only identify $\lambda_g + \frac{\beta_{0,g}}{1-\beta_{0,S}}\lambda_S$ and cannot separate λ_g from $\beta_{0,g}$.

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  2. Absent supply shocks, the only way to identify $\beta_{0,g}$ is via $u_S(n)$: Demand shocks of other investors that are uncorrelated with the demand shocks of investor group g.
- Classic examples:
  - Index inclusion: Shock to index investors.
  - Regulatory events: Shock to regulated investors (e.g., insurers).

Instrument (Version 1)

- Factor structure implies that portfolio weight for Apple depends
  - Directly on Apple's price and characteristics.
  - Indirectly on the characteristics of other stocks (e.g., Amazon) through market clearing.
- Instrument:

$$\widehat{\mathrm{me}}_i(n) = \log \left(\sum_{j \neq i} A_j \widehat{w}_j(n) \right)$$

  - $\widehat{w}_j(n)$ are predicted weights from a regression of portfolio weights onto characteristics only.

Instrument (Version 2)

$$\frac{w_i(n)}{w_i(0)} = \begin{cases} \mathbb{1}_i(n) \exp\left\{\beta_{0,i}\mathrm{me}(n) + \sum_{k=1}^{K} \beta_{k,i} x_k(n)\right\} \epsilon_i(n) & \text{if } n \in \mathcal{N}_i \\ \mathbb{1}_i(n) = 0 & \text{if } n \notin \mathcal{N}_i \end{cases}$$

- Investors may not hold an asset for two reasons.
  1. $\epsilon_i(n) = 0$: Chooses not to hold an asset.
  2. $\mathbb{1}_i(n) = 0$: Cannot hold an asset outside the investment universe.
- Assumption: Investment universe is exogenous.
- Instrument:

$$\widehat{\mathrm{me}}_i(n) = \log\left(\sum_{j \neq i} A_j \frac{\mathbb{1}_j(n)}{1 + \sum_{m=1}^{N} \mathbb{1}_j(m)}\right)$$

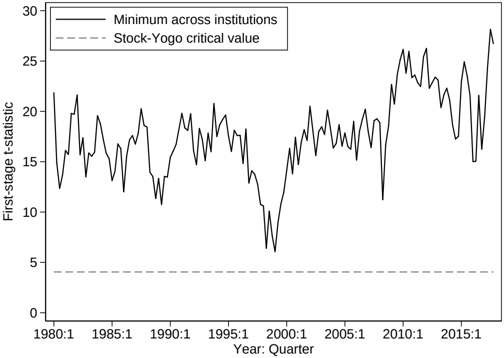
Intuition

- Index addition/deletion (e.g., Shleifer 1986) relates exogenous changes in demand to returns.
- Apply the same logic to the level of prices. Heterogeneous investment universe creates exogenous variation in demand that relates to price.
- Stocks that appear in the investment universe of more investors (weighted by AUM) has higher price.

Small number of assets in the portfolio

- For investors with at least 1,000 stocks in the portfolio, estimate coefficients individually.
- For investors with fewer stocks
  - Pooled estimation among investors of the same type and similar AUM (Koijen and Yogo 2019).
  - Ridge estimation by institution, shrinking toward the average coefficient for investors with at least 1,000 stocks (Koijen, Richmond, and Yogo 2019).

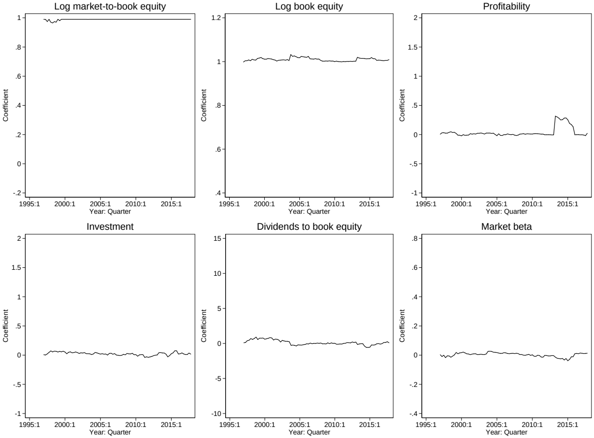
First-stage *t*-statistic on the instrument for log market equity



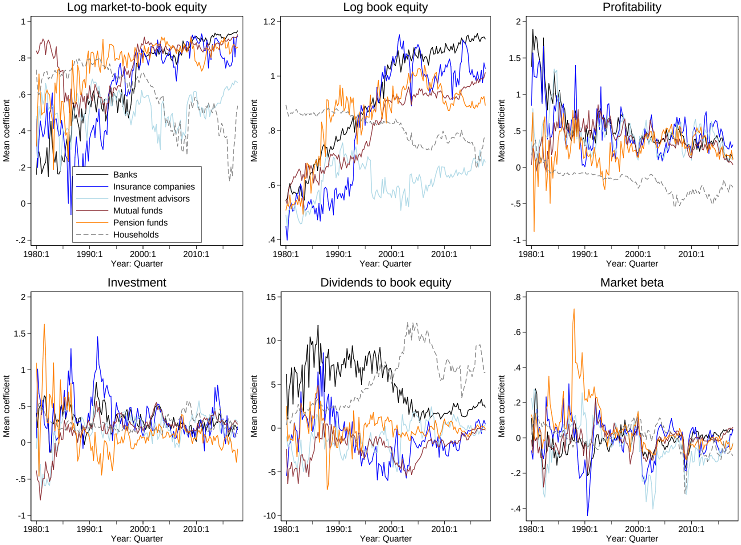
- Critical value for rejecting the null of weak instruments is 4.05 (Stock and Yogo 2005, Table 5.2).

Coefficients on characteristics for an index fund

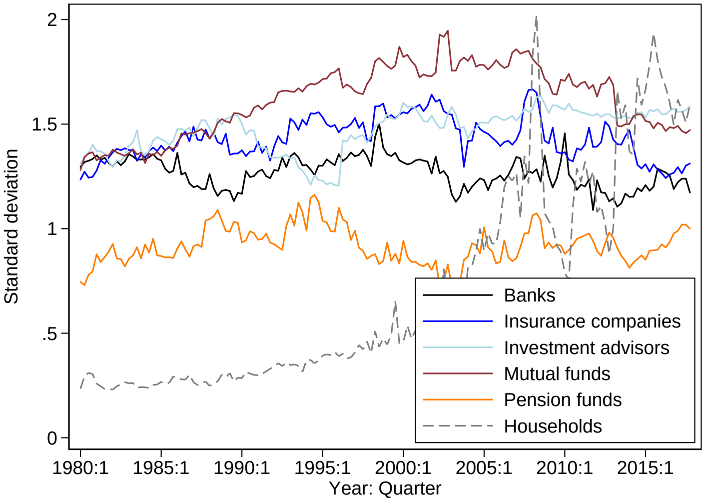
- A placebo test on an hypothetical index fund with market weights.



Coefficients on characteristics

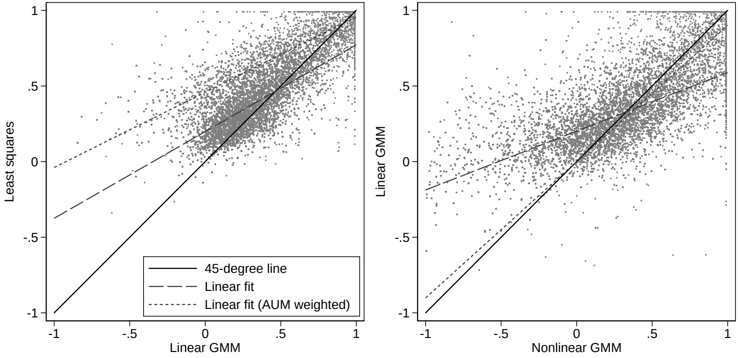


Standard deviation of latent demand



Comparison of the coefficients on log market equity

- Left: Least squares is upward biased.
- Right: Linear GMM (i.e., estimating in logs) is upward biased for smaller institutions.



Counterfactuals: Motivating questions

- We can use the asset demand system to compute counterfactuals.
- Examples of questions that can be explored:
  1. Have financial markets become more liquid over the last 30 years with the growing importance of institutional investors?
  2. How much of the volatility and predictability of asset prices is explained by institutional demand?
  3. Do large investment managers amplify volatility? Should they be regulated as SIFI?

Computing counterfactuals

- Recall the market clearing equation

$$ME(n) = S(n)P(n) = \sum_{i=1}^{I} A_i w_i(n, \mathbf{me}, \mathbf{x}, \epsilon).$$

- Taking logarithms implies

$$\mathbf{p} = \mathbf{f}(\mathbf{p}) = \log\left(\sum_{i=1}^{I} A_i \mathbf{w}_i(\mathbf{p})\right) - \mathbf{s}.$$

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- Market clearing defines an implicit function for log price:

$$\mathbf{p}_t = \mathbf{g}(\mathbf{s}_t, \mathbf{x}_t, \mathbf{A}_t, \beta_t, \epsilon_t).$$

$\Rightarrow$ Asset prices are fully determined by shares outstanding, characteristics, the wealth distribution, the coefficients on characteristics, and latent demand.

Computing counterfactuals

- To solve for prices, we need to solve a high-dimensional non-linear system.
- In practice, this can be done quite easily starting from the market clearing condition in logarithms:

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- Given a price vector $\mathbf{p}_m$, Newton's method would update the price vector through

$$\mathbf{p}_{m+1} = \mathbf{p}_m + \left(\mathbf{I} - \frac{\partial \mathbf{f}(\mathbf{p}_m)}{\partial \mathbf{p}'}\right)^{-1} (\mathbf{f}(\mathbf{p}_m) - \mathbf{p}_m).$$

- For our application, this approach would be computationally slow because the Jacobian has a large dimension.

Computing counterfactuals

- Therefore, we approximate the Jacobian with only its diagonal elements

$$\frac{\partial \mathbf{f}(\mathbf{p}_m)}{\partial \mathbf{p}'} \approx \operatorname{diag}\left(\min\left\{\frac{\partial f(\mathbf{p}_m)}{\partial p(n)}, 0\right\}\right)$$
$$= \operatorname{diag}\left(\min\left\{\frac{\sum_{i=1}^{I} \beta_{0,i} A_i w_i(\mathbf{p}_m; n)(1 - w_i(\mathbf{p}_m; n))}{\sum_{i=1}^{I} A_i w_i(\mathbf{p}_m; n)}, 0\right\}\right),$$

  where the minimum ensures that the elements are bounded away from one.

- We have found that this algorithm is fast and reliable, converging in fewer than 100 steps in applications.

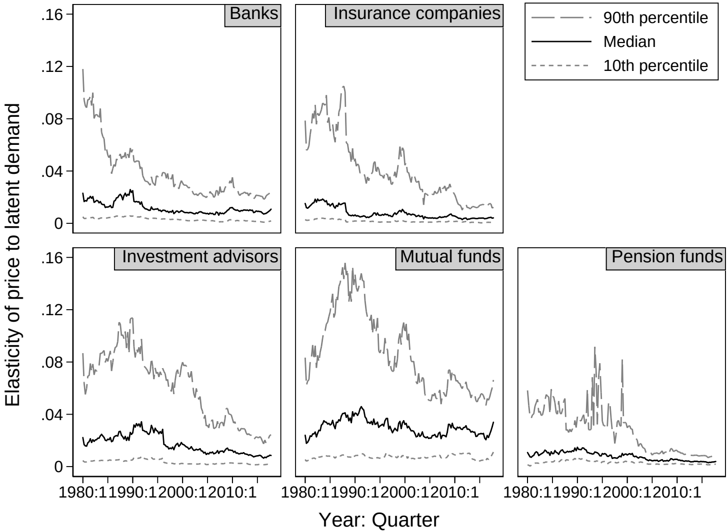
Liquidity measurement

- We define the co-liquidity matrix for investor i as

$$\frac{\partial \mathbf{p}_t}{\partial \log(\epsilon_{i,t})'} = \left(\mathbf{I} - \sum_{j=1}^{I} A_{j,t}\beta_{0,j,t}\mathbf{H}_t^{-1}\mathbf{G}_{j,t} \right)^{-1} A_{i,t}\mathbf{H}_t^{-1}\mathbf{G}_{i,t}.$$

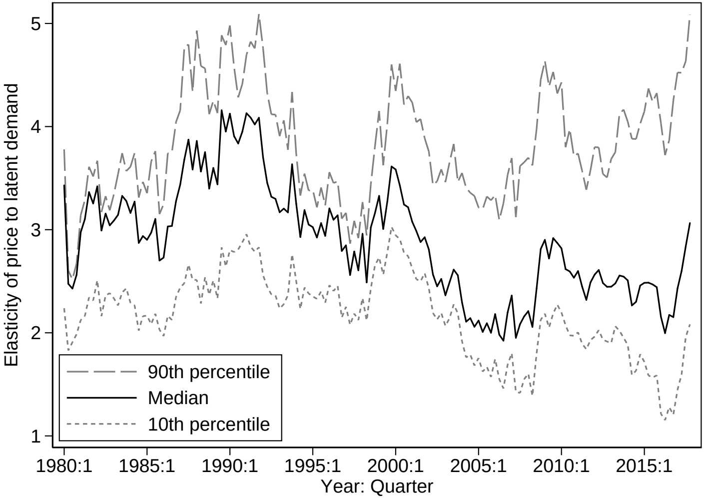
- We compute two measures of price impact
  - Price impact for each stock and institution via the diagonal elements of $\frac{\partial \mathbf{p}_t}{\partial \log(\epsilon_{i,t})'}$ and average by institutional type.
  - Aggregate price impact, defined as $\sum_{i=1}^{I} \frac{\partial \mathbf{p}_t}{\partial \log(\epsilon_{i,t})'}$, captures the price impact of systematic shocks to latent demand across all investors.

Price impact across stocks and institutions



Aggregate price impact across stocks

- Aggregate price impact: $\sum_{i=1}^{I} \partial p(n) / \partial \log(\epsilon_i(n))$.



Variance decomposition of stock returns

- We start with the definition of log returns:

$$\mathbf{r}_{t+1} = \mathbf{p}_{t+1} - \mathbf{p}_t + \mathbf{v}_{t+1},$$

where $\mathbf{v}_{t+1} = \log(\mathbf{1} + \exp\{\mathbf{d}_{t+1} - \mathbf{p}_{t+1}\})$.

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where $\mathbf{v}_{t+1} = \log(\mathbf{1} + \exp\{\mathbf{d}_{t+1} - \mathbf{p}_{t+1}\})$.

- The model implies that

$$\mathbf{p}_t = \mathbf{g}(\mathbf{s}_t, \mathbf{x}_t, \mathbf{A}_t, \beta_t, \epsilon_t)$$

1. $\mathbf{s}_t$: Shares outstanding.
2. $\mathbf{x}_t$: Asset characteristics.
3. $\mathbf{A}_t$: Assets under management.
4. β_t: Coefficients on characteristics.
5. ϵ_t: Latent demand.

Variance decomposition of stock returns

- We decompose the capital gain, $\mathbf{p}_{t+1} - \mathbf{p}_t$, as

$$\Delta\mathbf{p}_{t+1}(\mathbf{s}) + \Delta\mathbf{p}_{t+1}(\mathbf{x}) + \Delta\mathbf{p}_{t+1}(\mathbf{A}) + \Delta\mathbf{p}_{t+1}(\beta) + \Delta\mathbf{p}_{t+1}(\epsilon),$$

where:

$$\begin{aligned}\Delta\mathbf{p}_{t+1}(\mathbf{s}) =& \mathbf{g}(\mathbf{s}_{t+1}, \mathbf{x}_t, \mathbf{A}_t, \beta_t, \epsilon_t) - \mathbf{g}(\mathbf{s}_t, \mathbf{x}_t, \mathbf{A}_t, \beta_t, \epsilon_t),\\ \Delta\mathbf{p}_{t+1}(\mathbf{x}) =& \mathbf{g}(\mathbf{s}_{t+1}, \mathbf{x}_{t+1}, \mathbf{A}_t, \beta_t, \epsilon_t) - \mathbf{g}(\mathbf{s}_{t+1}, \mathbf{x}_t, \mathbf{A}_t, \beta_t, \epsilon_t),\\ & \ldots\end{aligned}$$

Variance decomposition of stock returns

- We decompose the capital gain, $\mathbf{p}_{t+1} - \mathbf{p}_t$, as

$$\Delta\mathbf{p}_{t+1}(\mathbf{s}) + \Delta\mathbf{p}_{t+1}(\mathbf{x}) + \Delta\mathbf{p}_{t+1}(\mathbf{A}) + \Delta\mathbf{p}_{t+1}(\beta) + \Delta\mathbf{p}_{t+1}(\epsilon),$$

where:

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- We compute each of these counterfactual price vectors and decompose the cross-sectional variance of log returns as

$$1 = \frac{\mathrm{Cov}(\Delta\mathbf{p}_{t+1}(\mathbf{s}), \mathbf{r}_{t+1})}{\mathrm{Var}(\mathbf{r}_{t+1})} + \frac{\mathrm{Cov}(\Delta\mathbf{p}_{t+1}(\mathbf{x}), \mathbf{r}_{t+1})}{\mathrm{Var}(\mathbf{r}_{t+1})} + \dots.$$

Variance decomposition of stock returns

	% of variance
Supply:	
Shares outstanding	2.1
	(0.2)
Stock characteristics	9.7
	(0.3)
Dividend yield	0.4
	(0.0)
Demand:	
Assets under management	2.3
	(0.1)
Coefficients on characteristics	4.7
	(0.2)
Latent demand: Extensive margin	23.3
	(0.3)
Latent demand: Intensive margin	57.5
	(0.4)
Observations	134,328

Variance decomposition of stock returns in 2008

- The asset demand system can also be used to understand how much an investor contributes to the the fluctuations in a given stock.
- This provides a new perspective on the "dark matter" in financial markets.

Variance decomposition of stock returns in 2008

- The asset demand system can also be used to understand how much an investor contributes to the the fluctuations in a given stock.
- This provides a new perspective on the "dark matter" in financial markets.
- We provide an illustration during the financial crisis.
- We modify the variance decomposition as

$$\begin{aligned}\mathrm{Var}(\mathbf{r}_{t+1}) = & \mathrm{Cov}(\Delta\mathbf{p}_{t+1}(\mathbf{s}) + \Delta\mathbf{p}_{t+1}(\mathbf{x}) + \mathbf{v}_{t+1}, \mathbf{r}_{t+1}) \\ & + \sum_{i=1}^{I} \mathrm{Cov}(\Delta\mathbf{p}_{t+1}(A_i) + \Delta\mathbf{p}_{t+1}(\beta_i) + \Delta\mathbf{p}_{t+1}(\epsilon_i), \mathbf{r}_{t+1}).\end{aligned}$$

Variance decomposition of stock returns in 2008

- Are large investment managers systemic?

AUM ranking	Institution	AUM ($ billion)	Change in AUM (%)	% of variance	
	Supply: Shares outstanding, stock characteristics & dividend yield			8.1	(1.0)
1	Barclays Bank	699	-41	0.3	(0.1)
2	Fidelity Management & Research	577	-63	0.9	(0.2)
3	State Street Corporation	547	-37	0.3	(0.0)
4	Vanguard Group	486	-41	0.4	(0.0)
5	AXA Financial	309	-70	0.3	(0.1)
6	Capital World Investors	309	-44	0.1	(0.1)
7	Wellington Management Company	272	-51	0.4	(0.1)
8	Capital Research Global Investors	270	-53	0.1	(0.1)
9	T. Rowe Price Associates	233	-44	-0.2	(0.1)
10	Goldman Sachs & Company	182	-59	0.1	(0.1)
	Subtotal: 30 largest institutions	6,050	-48	4.4	
	Smaller institutions	6,127	-53	40.7	(2.3)
	Households	6,322	-47	46.9	(2.6)
	Total	18,499	-49	100.0	

Predictability of stock returns

- Recall that

$$\mathbf{p}_T = \mathbf{g}(\mathbf{s}_T, \mathbf{x}_T, \mathbf{A}_T, \beta_T, \epsilon_T)$$

- Model ϵ_T as mean reverting and everything else as random walk.

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- Model ϵ_T as mean reverting and everything else as random walk.
- First-order approximation of expected long-run capital gain:

$$\begin{aligned}\mathbb{E}_t[\mathbf{p}_T - \mathbf{p}_t] \approx& \mathbf{g}(\mathbb{E}_t[\mathbf{s}_T], \mathbb{E}_t[\mathbf{x}_T], \mathbb{E}_t[\mathbf{A}_T], \mathbb{E}_t[\beta_T], \mathbb{E}_t[\epsilon_T]) - \mathbf{p}_t \\ =& \mathbf{g}(\mathbf{s}_t, \mathbf{x}_t, \mathbf{A}_t, \beta_t, \mathbf{1}) - \mathbf{p}_t\end{aligned}$$

- Intuition: Assets with high latent demand are expensive and have low expected returns.

Relation between stock returns and characteristics

Characteristic	All stocks	Excluding microcaps
Expected return	0.18	0.11
	(0.04)	(0.04)
Log market equity	-0.25	-0.15
	(0.08)	(0.08)
Book-to-market equity	0.04	0.06
	(0.04)	(0.05)
Profitability	0.30	0.29
	(0.06)	(0.06)
Investment	-0.38	-0.21
	(0.03)	(0.03)
Market beta	0.08	0.01
	(0.08)	(0.10)
Momentum	0.24	0.37
	(0.08)	(0.10)

Predicting returns by predicting demand

- Modern approaches to return predictability take characteristics of stocks and use it to predict returns directly.
- DSAP provides another approach by first predicting demand and then predict returns via market clearing.
- The conditions under which both approaches are equivalent are quite strong and require a lot of homogeneity across investors.
- Predicting returns by predicting demand can yield new insights by taking a more granular approach. We just provide a simple first example as a "proof of concept."

Additional applications

- In Koijen, Richmond, and Yogo (2023), we provide additional examples of counterfactuals:
  - How important are different investors for pricing characteristics (e.g., governance or environmental characteristics).
  - How did the transition from active to passive investment affect prices, investors' wealth, and price informativeness?
  - Climate stress tests: If there is a shift in demand for green characteristics (e.g., because of growing awareness or because of new regulation for insurers and pension funds), how would this affect prices and investors' wealth.

Lucas critique

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- Also, we cannot answer welfare questions without taking an explicit stance on preferences, beliefs, and constraints.
- However, for most asset pricing applications, price (rather than welfare) is the primary object of interest.
- That said, it highlights the importance of developing new micro foundations that can deliver inelastic demand and other key features of the asset demand system.

Conclusions

- We show how to calculate counterfactuals once we have estimated the asset demand system.
- The demand system can be use to connect fluctuations in prices to changes in characteristics and investors' demand.
- This provides a new perspective to start analyzing the "dark matter" in financial markets.
- Moreover, by predicting demand, we provide a new approach to return predictability, where machine learning/AI methods are particularly well suited as holdings data are very high-dimensional.