Demand System Asset Pricing
Estimating Asset Demand Systems

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Summary of Week 1

- Every asset pricing model that imposes market clearing ***implies an asset demand system.***
- DSAP explores the models’ predictions in terms of this implied asset demand system using holdings data.
  - In addition to the standard data on prices, characteristics, and fundamentals.
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  - In addition to the standard data on prices, characteristics, and fundamentals.
- Why is a well-specified asset demand system important?
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- Every asset pricing model that imposes market clearing implies an asset demand system.
- DSAP explores the models’ predictions in terms of this implied asset demand system using holdings data.
  - In addition to the standard data on prices, characteristics, and fundamentals.
- Why is a well-specified asset demand system important?
  - Many questions are “quantity questions.”
    - Trend from active to passive investing, ESG investing, . . .
    - The demand for safe assets and the convenience yield on US securities.
    - The impact of policy on asset prices, e.g., QE, risk regulation, fiscal capacity, . . .

To obtain credible answers, we need a quantitatively realistic model of the asset demand system.
Summary of Week 1

- How well do standard models do? Theoretical predictions:
  - Macro elasticity (stocks versus bonds): $\approx 20$.

- Empirical estimates using different instruments, different countries, and different levels of aggregation find that micro elasticities are around 1 and macro elasticities below that.

- Key takeaway:
  Models imply asset demand curves that are far too elastic.
Summary of Week 1

- We discussed a micro foundation of an empirically-tractable asset demand system.
- The portfolio weight on stock $n$ is

$$w_i(n) = \frac{\delta_i(n)}{1 + \sum_{m \in N_i} \delta_i(m)},$$

where

$$\delta_i(n) = \exp(b_{0,i} + \beta_{0,i}me(n) + \beta_{1,i}'x(n))\epsilon_i(n).$$

- Today, we will discuss:
  1. Model estimation and identification.
  2. Computing counterfactuals.
  3. Applications to liquidity measurement, understanding the “dark matter” of financial markets, and return predictability.
Identification and estimation of asset demand systems

Two central issues in asset demand estimation:

1. Latent demand is jointly endogenous with asset prices.
   - This is true when some investors are large or when latent is correlated across investors.
   - We need an instrument to estimate the model.

2. Implementation choices.
   - Some investors hold concentrated portfolios.
   - How to handle zero holdings in investors’ portfolios.
Empirical specification

- Our model for demand

\[ w_i(n) = \frac{\delta_i(n)}{1 + \sum_{m \in \mathcal{N}_i} \delta_i(m)}, \]

implies for the fraction invested in the outside asset

\[ w_i(0) = 1 - \sum_{n \in \mathcal{N}_i} w_i(n) = \frac{1}{1 + \sum_{m \in \mathcal{N}_i} \delta_i(m)}. \]
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- Combining both equations implies

\[ \frac{w_i(n)}{w_i(0)} = \delta_i(n) = \exp(b_{0,i} + \beta_{0,i} me(n) + \beta'_{1,i} x(n)) \epsilon_i(n). \]
Empirical specification

Given an instrument for market cap, $\hat{me}_i(n)$, we can estimate the model in two ways:

1. Nonlinear GMM (with zero weights).

$$\frac{w_i(n)}{w_i(0)} = \exp(b_{0,i} + \beta_{0,i} me(n) + \beta_{1,i}'x(n))\epsilon_i(n)$$

- **Moment condition:** $E[\epsilon_i(n)|\hat{me}_i(n), x(n)] = 1$. 
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   \]

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2. Linear IV (without zero weights).

   \[
   \log \left( \frac{w_i(n)}{w_i(0)} \right) = b_{0,i} + \beta_{0,i}me(n) + \beta'_{1,i}x(n) + \log(\epsilon_i(n))
   \]

   - **Moment condition:** $E[\log(\epsilon_i(n))|\hat{me}_i(n), x(n)] = 0$. 
Empirical specification

- Characteristics.
  2. Profitability.
  3. Investment.
  5. Market beta.

- For each 13F institution and the household sector, use the cross-section of holdings to estimate coefficients at each point in time.

- Traditional assumption in endowment economies:

\[
\mathbb{E}[\epsilon_i(n)|me(n), x(n)] = 1
\]
The role of the outside asset in estimation

- When we include investor-quarter fixed effects in the specification, $a_{it}$, the choice of the outside asset does not matter for estimation:

\[
\log \left( \frac{w_{it}(n)}{w_{it}(0)} \right) = a_{it} + \beta_{0,it}m_{et}(n) + \beta'_{1,it}x_{t}(n) + \log(\epsilon_{it}(n)).
\]

- Any choice of $w_{it}(0)$ will be absorbed in $a_{it}$ and we can equivalently estimate:

\[
\log (w_{it}(n)) = (a_{it} + \log(w_{it}(0)) + \beta_{0,it}m_{et}(n) + \beta'_{1,it}x_{t}(n) + \log(\epsilon_{it}(n)).
\]

- The choice of the outside asset will matter in counterfactuals.
Identification

- Latent demand is generally correlated with prices.
  - Mechanically true if an investor is large.
  - Even with a continuum of investors, if there are common components in latent demand (e.g., sentiment, news media, corporate events, ...), then latent demand and prices are correlated.

- We therefore need an instrument for market equity.

- Before discussing specific instruments, we develop some intuition for where to find candidate instruments.
Identification: Intuition

- Portfolio weight for a group of investors indexed by $g$ (omitting constants)

$$w_g(n) = \beta_{0,g} ME(n) + \lambda_g \eta(n) + u_g(n),$$

where $u_g(n)$ is uncorrelated across groups of investors.
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  where \( u_g(n) \) is uncorrelated across groups of investors.

- Market clearing implies \( \sum_g A_g w_g(n) = ME(n) \) and thus
  \[
  ME(n) = \frac{\lambda_S \eta(n) + u_S(n)}{1 - \beta_{0,S}},
  \]
  where \( x_S = \frac{\sum_g A_g x_g}{\sum_g A_g} \), the size-weighted average.
Identification: Intuition

- Substitute the market clearing price into the demand equation

\[ w_g(n) = \left( \lambda_g + \frac{\beta_{0,g}}{1 - \beta_{0,S}} \lambda_S \right) \eta(n) + \frac{\beta_{0,g}}{1 - \beta_{0,S}} u_S(n) + u_g(n). \]
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- Key insights:
  1. Common demand shocks, \( \eta(n) \), cannot be used to identify elasticities. We can only identify \( \lambda_g + \frac{\beta_{0,g}}{1 - \beta_{0,s}} \lambda_S \) and cannot separate \( \lambda_g \) from \( \beta_{0,g} \).
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2. Absent supply shocks, the only way to identify \( \beta_{0,g} \) is via \( u_S(n) \): Demand shocks of other investors that are uncorrelated with the demand shocks of investor group \( g \).
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- Classic examples:

  - Index inclusion: Shock to index investors.
  - Regulatory events: Shock to regulated investors (e.g., insurers).
Instrument (Version 1)

- Factor structure implies that portfolio weight for Apple depends
  - Directly on Apple’s price and characteristics.
  - Indirectly on the characteristics of other stocks (e.g., Amazon) through market clearing.

- Instrument:

\[
\widehat{m}_{ei}(n) = \log \left( \sum_{j \neq i} A_j \widehat{w}_j(n) \right)
\]

- \(\widehat{w}_j(n)\) are predicted weights from a regression of portfolio weights onto characteristics only.
Instrument (Version 2)

\[
\frac{w_i(n)}{w_i(0)} = \begin{cases} \mathbb{1}_i(n) \exp \left\{ \beta_{0,i} \text{me}(n) + \sum_{k=1}^{K} \beta_{k,i} x_k(n) \right\} \epsilon_i(n) & \text{if } n \in \mathcal{N}_i \\ \mathbb{1}_i(n) = 0 & \text{if } n \notin \mathcal{N}_i \end{cases}
\]

- Investors may not hold an asset for two reasons.
  1. \(\epsilon_i(n) = 0\): Chooses not to hold an asset.
  2. \(\mathbb{1}_i(n) = 0\): Cannot hold an asset outside the investment universe.

- Assumption: Investment universe is exogenous.

- Instrument:

\[
\hat{\text{me}}_i(n) = \log \left( \sum_{j \neq i} A_j \frac{\mathbb{1}_j(n)}{1 + \sum_{m=1}^{N} \mathbb{1}_j(m)} \right)
\]
Intuition

- Index addition/deletion (e.g., Shleifer 1986) relates exogenous changes in demand to returns.
- Apply the same logic to the level of prices. Heterogeneous investment universe creates exogenous variation in demand that relates to price.
- Stocks that appear in the investment universe of more investors (weighted by AUM) has higher price.
Small number of assets in the portfolio

- For investors with at least 1,000 stocks in the portfolio, estimate coefficients individually.
- For investors with fewer stocks
  - Pooled estimation among investors of the same type and similar AUM (Koijen and Yogo 2019).
  - Ridge estimation by institution, shrinking toward the average coefficient for investors with at least 1,000 stocks (Koijen, Richmond, and Yogo 2019).
First-stage $t$-statistic on the instrument for log market equity

- Critical value for rejecting the null of weak instruments is 4.05 (Stock and Yogo 2005, Table 5.2).
Coefficients on characteristics for an index fund

- A placebo test on an hypothetical index fund with market weights.

![Graphs showing coefficients for various characteristics over time.](image)
Coefficients on characteristics

- Log market-to-book equity
- Log book equity
- Profitability
- Investment
- Dividends to book equity
- Market beta
Standard deviation of latent demand

![Standard deviation of latent demand graph](image-url)
Comparison of the coefficients on log market equity

- Left: Least squares is upward biased.
- Right: Linear GMM (i.e., estimating in logs) is upward biased for smaller institutions.
Conclusion

- Research in empirical asset pricing has uncovered factor structure in returns, and expected returns and factor loadings that relate to an asset’s own characteristics.
- This key fact, when applied to portfolio choice, leads to a strategy for identification of asset demand systems.
- In micro data with institutional holdings, portfolio concentration through investment mandates gives us a weaker identification assumption.
- As ETF’s and index strategies become bigger, opportunities to measure the investment mandate more accurately and further refine the instrument in KY19.